NArcs and FArcs

Near Arcs and Far Arcs

<u>Narcs and Farcs</u>

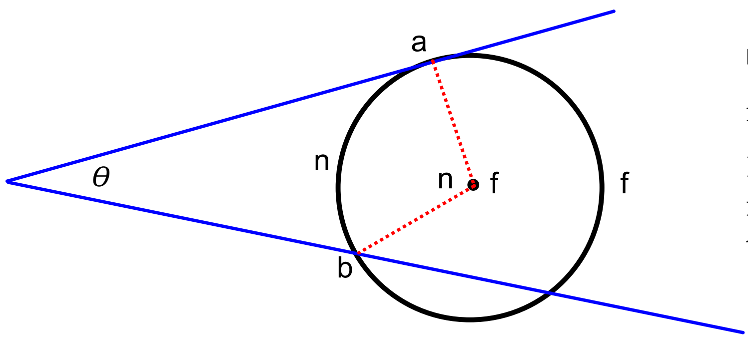
Near Arcs and Far Arcs.

These are ratios that are cause by the intersection of segments or lines with a circle.

The ratios come about because of <u>arc measures</u> and/or <u>similar triangles</u>.

Similarity can be proved using arc measures.

In these slides f and n refer to the near arc and far arc.



n is the arc nearest $\angle\theta$

f is the arc farthest $\angle\theta$

Remember the arc measure is the same as the central angle

There are three scenarios that you need to recognize and work with. In each scenario there is a ratio of angles and a ratio of sides.

The slides follow the same order.

First I will show you the angle ratio,

then I will show you the proof of the angle ratio,

then will be an example.

You do not need to remember the proofs, though knowing them will help you solve complex problems.

The ratio of sides comes from triangle similarity. Which you should be familiar with. Review if necessary.

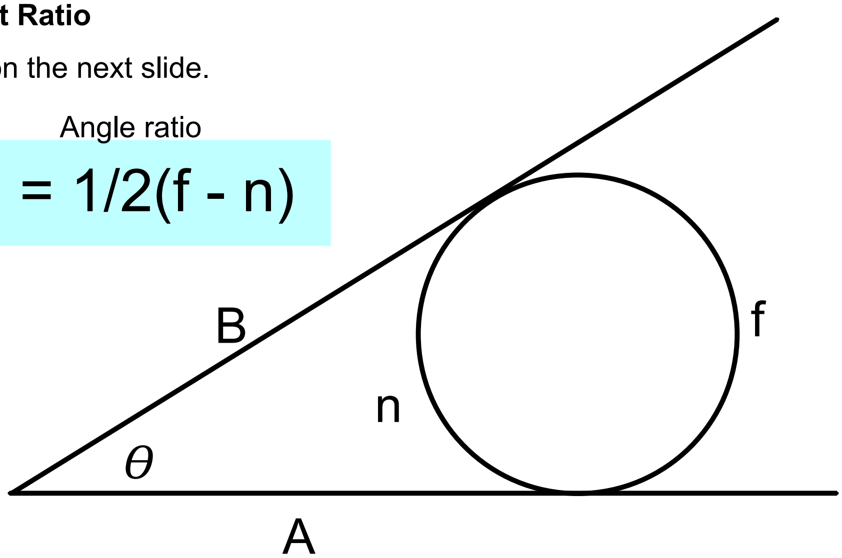
Applying a formula to solve a problem is at most C level work.

The First Ratio

Proved on the next slide.

Angle ratio

$\theta = 1/2(f - n)$



Side ratio

A = B

A and B are both tangents

First Ratio - proof

arcs *f* and *n* add to 360° because they make the circle.

so $f + n = 360$

abcd make a quadrilateral

so $\theta + 90 + 90 + n = 360$ (QAST)

$\theta + n = 180$ *(simplify)*

$2(\theta + n) = 360$ *(mult both sides by 2)*

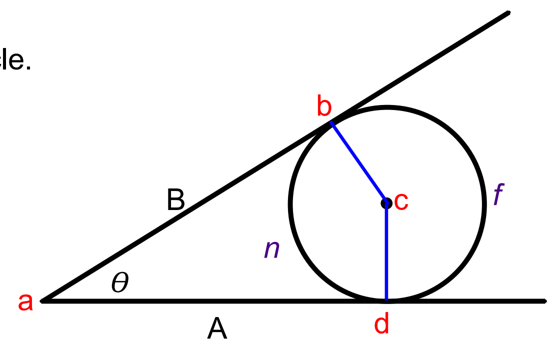
$f + n = 2(\theta + n)$ *(set equations equal to each other)*

$f + n = 2\theta + 2n$ *(distribute the 2)*

$f - n = 2\theta$ *(subtract 2n from both sides)*

$1/2(f - n) = \theta$ *(divide both sides by 2)*

$\theta = \frac{1}{2}(f - n)$ *(rearrange to solve for θ)*



A = B

They are both tangents,

so ∠b and ∠d are right.

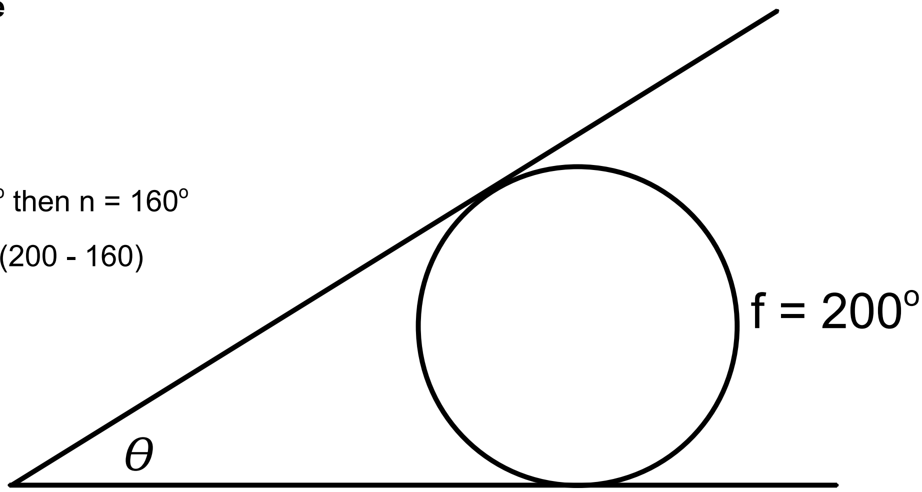
Example

Find θ.

if f = 200^o then n = 160^o

so θ = ½(200 - 160)

<u>θ = 20^o</u>



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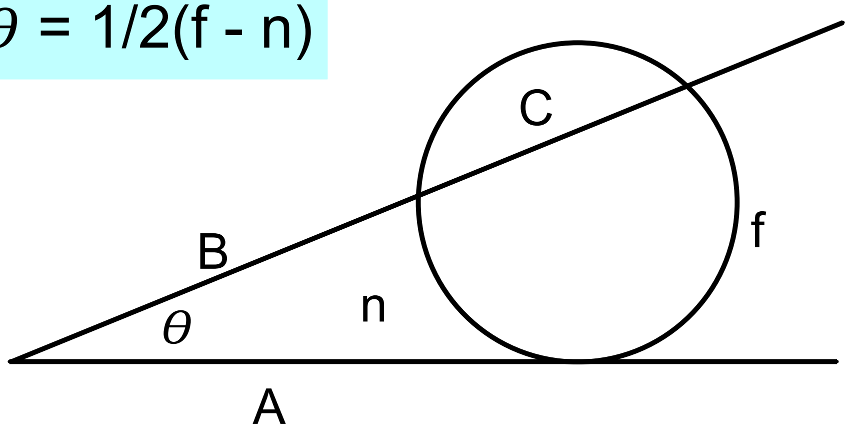
The Second Ratio

Angle ratio

$$\theta = 1/2(f - n)$$



Side ratio

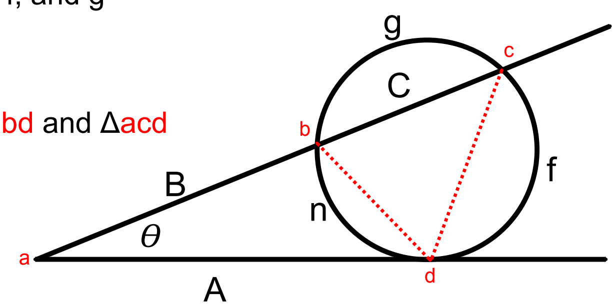
$$A^2 = B(B + C)$$

A is a tangent

Second ratio - proof

First we label the three arcs that make the circle n, f, and g

so $f + n + g = 360^{\circ}$



Connecting the points of intersection we create Δabd and Δacd

We're only going to use Δacd

∠acd subtends arc n so m∠acd = ½n

∠adc subtends arc n and g so m∠adc = ½(n+g)

so ∠acd + ∠adc + θ = 180o (ΔAST using Δacd)

$\frac{1}{2}(n+g) + \frac{1}{2}n + \theta = 180$ *(substituting arc measures)*

$\frac{1}{2}g + n + \theta = 180$ *(distribution and simplifying)*

$g + 2n + 2\theta = 360$ *(multiply both sides by 2)*

$f + n + g = g + 2n + 2\theta$ *(set both equations equal to each other)*

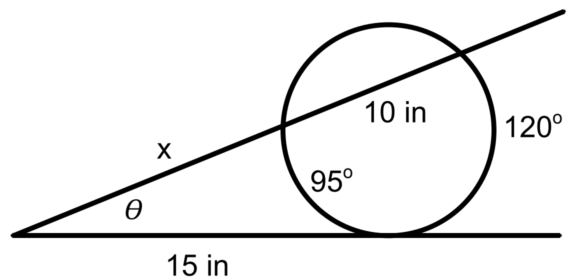
$f - n = 2\theta$ *(subtract 2n and g from both sides)*

θ **= (f-n)** *(rearranging to solve for θ)*

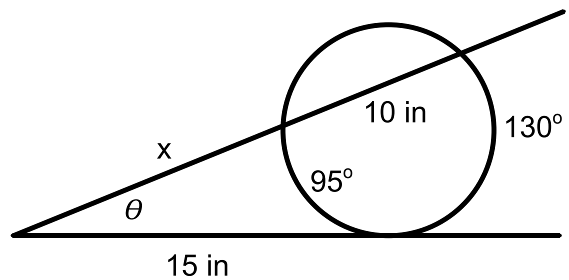
Example

Find θ and x.

Do it yourself first, then check you answers on the next page.



Find θ and x.



$\theta = \frac{1}{2}(130 - 95)$

$\theta = \frac{1}{2}(35)$

$\boldsymbol{\theta = 17.5^o}$

$15^2 = x(x + 10)$

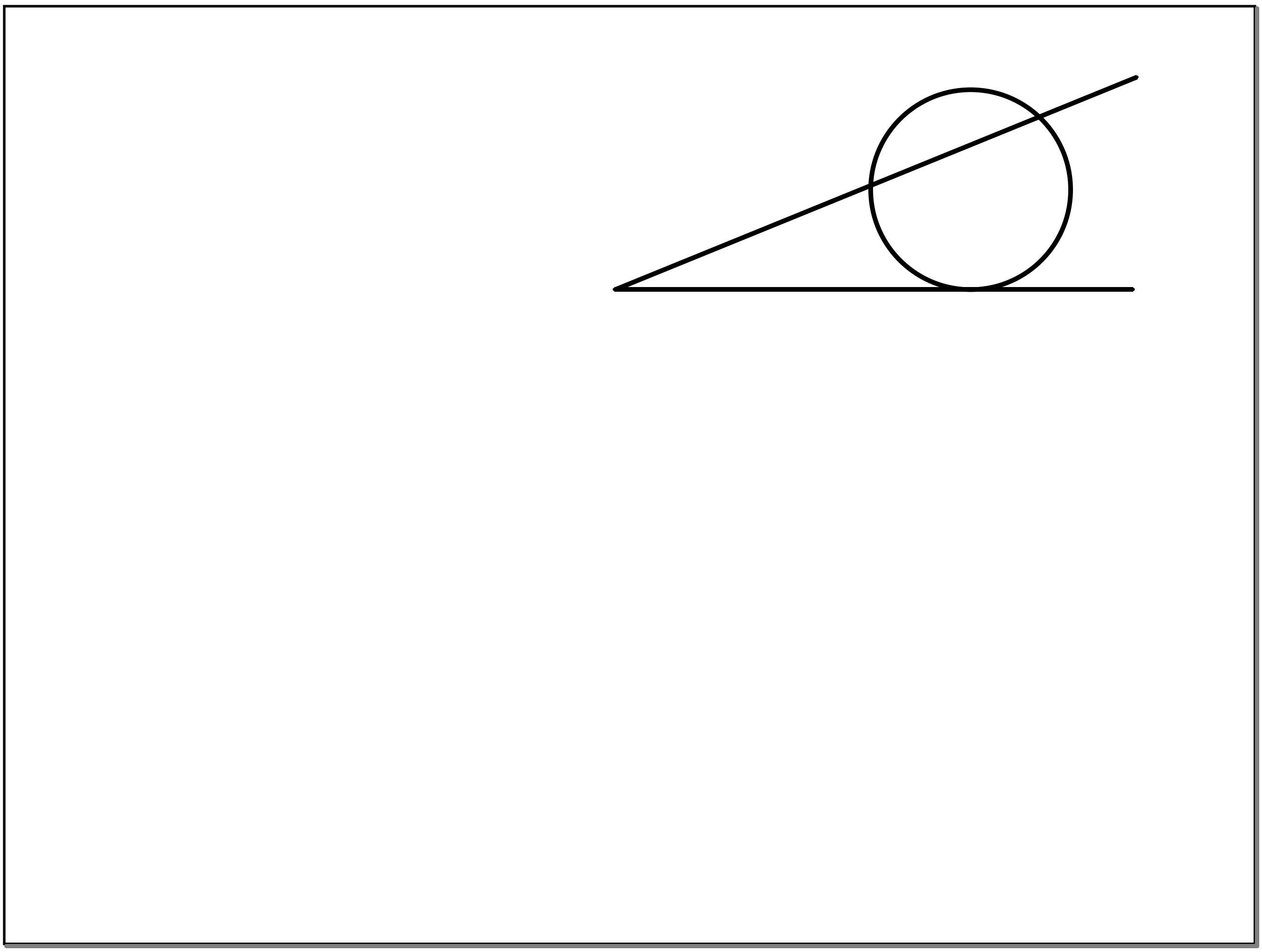
$225 = x^2 + 10x$

$0 = x^2 + 10x - 225$

x = -20.811, 10.811 (by quadratic formula)

a distance cannot be negative,

so **$x \approx 10.8$**



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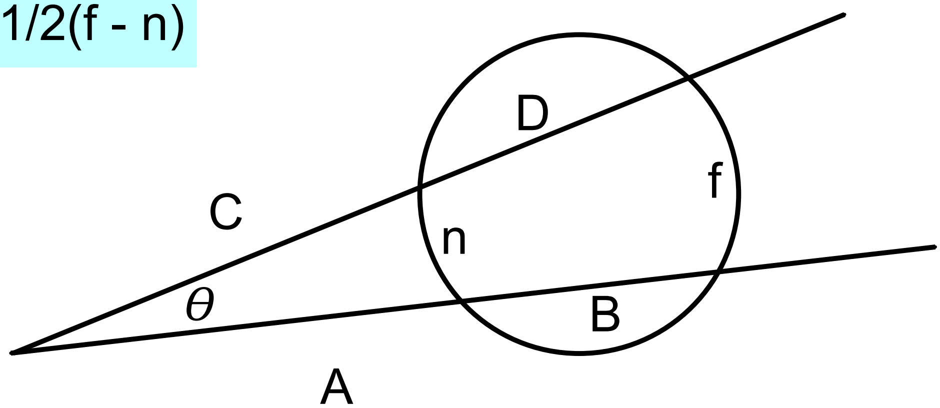
<u>The Third Ratio</u>

Angle ratio

$\theta = 1/2(f - n)$



Side ratio

$A(A + B) = C(C + D)$

The circle is made up of the arcs g, f, h, and n

so $g + f + h + n = 360^o$

Marking the intersection points gives us two triangles Δabe and Δadc

We are going to use Δadc

$\angle adc$ subtends the arcs g and n so $m\angle adc = ½(g + n)$

$\angle acd$ subtends the arcs n and h so $m\angle acd = ½(n + h)$

from Δacd

$\theta + ½(g + n) + ½(n + h) = 180$ *(ΔAST using Δacd)*

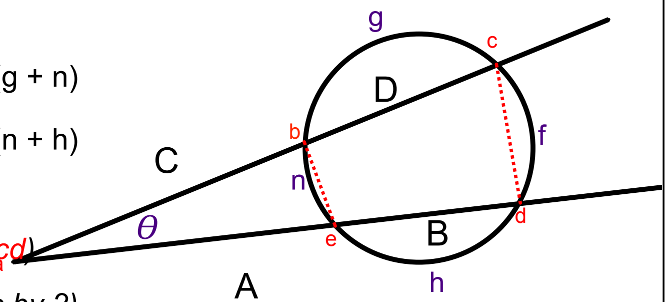
so $2\theta + g + n + n + h = 360$ *(mutliply both sides by 2)*

$2\theta + g + 2n + h = 360$ *(simplify)*

$g + f + h + n = 2\theta + g + 2n + h$ *(set two equations equal to each other)*

$f - n = 2\theta$ *(subtract g, h, and 2n from both sides)*

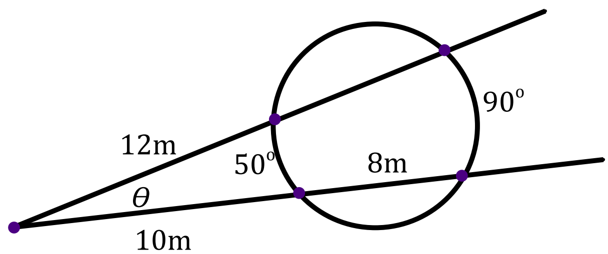
$\boldsymbol{\theta = ½(f - n)}$ *(rearrange to solve for θ)*



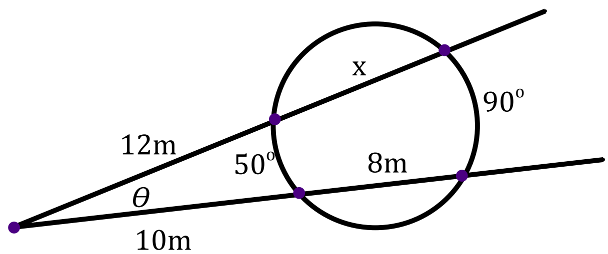
Example

Find θ and x.

Do it yourself first, then check you answers on the next page.



Find θ and x.



$\theta = ½(90 - 50)$

$\theta = ½(40)$

<u>θ **= 20°**</u>

$10(10 + 8) = 12(12 + x)$

$180 = 144 + 12x$

<u>**x = 3m**</u>

fin

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